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# Coupled Parallel Flow of Fluids with Viscosity Stratification through Composite Porous Layers

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***Abstract:-*** Coupled, parallel flow of fluids with viscosity stratification through two porous layers is initiated in this work. Conditions at the interface are discussed and appropriate viscosity stratification functions are selected in such a way that viscosity is highest at the bounding walls and decreases to reach its minimum at the interface. Velocity and shear stress at the interface are computed for different permeability and driving pressure gradient.

***Keywords:-*** *Coupled parallel flow, stratified viscosity*.

## I. INTRODUCTION

Coupled parallel flow through a channel bounded by a porous layer has received considerable attention in the literature due to the importance of this flow in physical applications. These range from ground water recovery and oil and gas production, to lubrication mechanism design and the design of heating and cooling systems. Conditions at the interface between the flow domains were first formalized by Beavers and Joseph, [1], whose experimental and theoretical work was based on Navier-Stokes flow in the channel and Darcy's flow in the porous layer. Much has been accomplished in this field since the introduction of Beavers and Joseph condition. In fact, various models of flow through porous media have been implemented in the analysis of interfacial conditions, and many excellent advancements in the field have contributed significantly to our state of knowledge. A number of excellent reviews of the problem, the models, and the knowledge base, are available (*cf.* [2-10] and the references therein).

Associated with the above coupled parallel flow is the flow through two or more porous layers. This situation was identified by Thiagaraja & Vafai [2] as the second problem of interest in this field. This situation arises in physical flows such as the flow of ground water in the (essentially) layered soil and bedrock, in agriculture and irrigation problems, and in biomedical applications involving flow of blood in animal tissues. The literature reports on a fairly large number of contributions in this field (*cf.* [11] and the references therein).

While the study of fluid flow with variable viscosity in general represents a rather established field of work, [12], and variations of viscosity are attributed to temperature and pressure, the flow of fluids with variable viscosity through porous structures is, by comparison, less established. Applications of this type of flow are found in the oil and pharmaceutical industries, among others, [13-21], and various flow models and problems have been discussed in the pioneering work of Rajagopal and coworkers (*cf.* [13-14], [18-20] and the references therein). Less studied, however, is the coupled parallel flow where permeability of the porous medium and viscosity of the fluid vary with position and flow conditions. In the current work, coupled parallel flows of fluids with variable, stratified viscosities through a composite of porous layers is considered. Matching conditions at the interface between layers are discussed and applied to a flow situation in which viscosities are stratified and vary with position. There is no assumption that viscosity variations are due to pressure or temperature variations in the current work. However, we set the stage for future considerations of the flow of fluids with pressure-dependent viscosities.

## II. PROBLEM FORMULATION

Consider the flow in the configuration shown in **Fig. 1**.

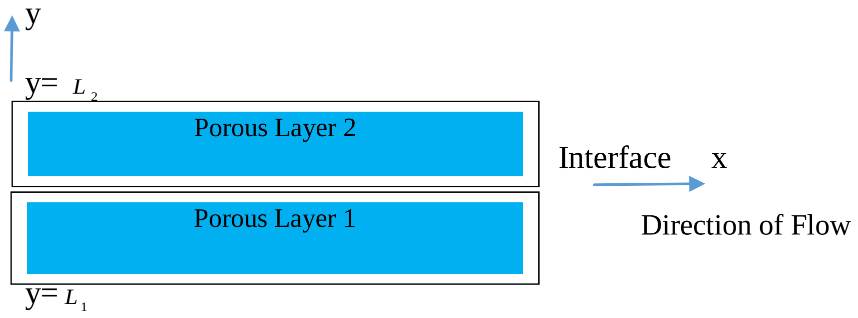


**Fig. 1. Representative Sketch**

Porous layer 1 is described by $\{(x,y)|x \geq 0, L_1 < y < 0\}$, and is bounded by an impermeable wall at $y = L_1$, while porous layer 2 is described by $\{(x,y)|x \geq 0, 0 < y < L_2\}$, and is bounded by an impermeable wall at $y = L_2$. An assumed sharp, permeable interface, exists between the two layers along the line *y = 0*. The fluids saturating the layers possess stratified, variable viscosities, and their motions are described by the following equations, for layers 1 and 2, respectively, and are based on model equations derived by Alharbi et al., [12]:

$$\mu_1 \frac{d^2 u_1}{dy^2} + \frac{d\mu_1}{dy}\frac{du_1}{dy} - \frac{\mu_1}{k_1} u_1 = p_{1x} \qquad \ldots(1)$$

$$\mu_2 \frac{d^2 u_2}{dy^2} + \frac{d\mu_2}{dy}\frac{du_2}{dy} - \frac{\mu_2}{k_2} u_2 = p_{2x} \qquad \ldots(2)$$

where, for layer *j* = 1,2, $\mu_j = \mu_j(y_j)$ is the variable viscosity of the fluid, $u_j = u_j(y)$ is the velocity of the fluid, $p_{jx} = C < 0$ is the constant, common, driving pressure gradient, and $k_j$ is the permeability.

Equations (1) and (2) are to be solved subject to the following boundary and interfacial conditions:

*No-slip condition at the impermeable walls:*

$$u_1(L_1) = u_2(L_2) = 0\,. \qquad \ldots(3)$$

*Conditions at the interface y = 0:*

At the interface, the velocity, shear stress, and viscosity of the fluid are assumed continuous. Therefore, we have:

$$u_1(0^-) = u_2(0^+) \qquad \ldots(4)$$

$$\mu_1 \frac{du_1}{dy}\bigg|_{y=0^-} = \mu_2 \frac{du_1}{dy}\bigg|_{y=0^+} \qquad \ldots(5)$$

$$\mu_1\Big|_{y=0^-} = \mu_2\Big|_{y=0^+}\,. \qquad \ldots(6)$$

It is assumed that the viscosity in each layer is stratified in the y-direction, hence a function of y only, and is of the exponential forms:

$$\mu_1(y) = \mu_0 \exp(\alpha_1 f_1) \qquad \ldots(7)$$

$$\mu_2(y) = \mu_0 \exp(\alpha_2 f_2) \qquad \ldots(8)$$

where $\mu_0$ is a constant reference viscosity (which could be taken as here as the fluid viscosity at atmospheric pressure and room temperature), $\alpha_1$ and $\alpha_2$ are specified constants, and the functions $f_1(y)$ and $f_2(y)$ are to be specified.

Now, using the chain rule, we obtain

$$\frac{d\mu_1}{dy} = \frac{d\mu_1}{df_1}\frac{\partial f_1}{\partial y} \qquad \ldots(9)$$

$$\frac{d\mu_2}{dy} = \frac{d\mu_2}{df_2}\frac{\partial f_2}{\partial y} \qquad \ldots(10)$$

and upon substituting (9) and (10) in (1) and (2), we obtain the following governing equations:

$$\frac{d^2u_1}{dy^2} + [\alpha_1 \frac{df_1(y)}{dy}]\frac{du_1}{dy} - \frac{u_1}{k_1} = \frac{C}{\mu_0}\exp[-\alpha_1 f_1(y)] \qquad \ldots(11)$$

$$\frac{d^2u_2}{dy^2} + [\alpha_2 \frac{df_2(y)}{dy}]\frac{du_2}{dy} - \frac{u_2}{k_2} = \frac{C}{\mu_0}\exp[-\alpha_2 f_2(y)] \, . \qquad \ldots(12)$$

Once the functions $f_1(y)$ and $f_2(y)$ are specified, we can then solve (11) and (12) for $u_1$ and $u_2$.

## III. SOLUTION METHODOLOGY

When the permeabilities $k_1$ and $k_2$ are constant, and $f_1(y)$ and $f_2(y)$ are given by

$$f_1(y) = a_1 y + b_1 \qquad \ldots(13)$$

$$f_2(y) = a_2 y + b_2 \qquad \ldots(14)$$

then the viscosity expressions (7) and (8) take the forms:

$$\mu_1(y) = \mu_0 \exp(\alpha_1[a_1 y + b_1]) \qquad \ldots(15)$$

$$\mu_2(y) = \mu_0 \exp(\alpha_2[a_2 y + b_2]) \qquad \ldots(16)$$

where $a_1, a_2, b_1, b_2$ are constants to be specified or to be determined. Viscosity at the interface is given by the following expressions:

$$\mu_1(0) = \mu_0 \exp(\alpha_1 b_1) \qquad \ldots(17)$$

$$\mu_2(0) = \mu_0 \exp(\alpha_2 b_2) \qquad \ldots(18)$$

which, upon invoking condition (6) and simplifying, yield the following relationship between the constants $\alpha_1, \alpha_2, b_1$ and $b_2$:

$$\alpha_1 b_1 = \alpha_2 b_2 \, . \qquad \ldots(19)$$

In order to determine the effects of the porous layer thicknesses, we first find a relationship between the thicknesses and the constants that determine the forms of the viscosity expressions. This is accomplished by letting $\mu_L$ and $\mu_u$ be the viscosities of the fluids at the lower and upper walls, respectively, namely:

$$\mu_1(L_1) = \mu_L = \mu_0 \exp(\alpha_1[a_1 L_1 + b_1]) \qquad \ldots(20)$$

$$\mu_2(L_2) = \mu_u = \mu_0 \exp(\alpha_2[a_2 L_2 + b_2]) \, . \qquad \ldots(21)$$

Now, taking $\mu_u = \mu_L$ at the walls, and using (19), equations (20) and (21) yield:

$$\alpha_1 a_1 L_1 = \alpha_2 a_2 L_2 \, . \qquad \ldots(22)$$

Equations (19) and (22) must be satisfied by the choice of coefficients in the viscosity functions (15) and (16). The governing equations (11) and (12) thus take the forms, which must be solved for $u_1$ and $u_2$:

$$\frac{d^2u_1}{dy^2} + \alpha_1 a_1 \frac{du_1}{dy} - \frac{u_1}{k_1} = \frac{C}{\mu_0}\exp[-\alpha_1(a_1 y + b_1)] \, . \qquad \ldots(23)$$

$$\frac{d^2u_2}{dy^2} + \alpha_2 a_2 \frac{du_2}{dy} - \frac{u_2}{k_2} = \frac{C}{\mu_0}\exp[-\alpha_2(a_2 y + b_2)] \, . \qquad \ldots(24)$$

General solutions to (23) and (24) take the following forms, respectively

$$u_1 = c_1 \exp(m_1 y) + c_2 \exp(m_2 y) - \frac{C}{\mu_0} k_1 \exp((m_1 + m_2)y - \alpha_1 b_1) \qquad \ldots(25)$$

$$u_2 = c_3 \exp(n_1 y) + c_4 \exp(n_2 y) - \frac{C}{\mu_0} k_2 \exp((n_1 + n_2)y - \alpha_2 b_2) \qquad \ldots(26)$$

where $c_1$ , $c_2$ , $c_3$ , $c_4$ are arbitrary constants, and

$$m_1 = \frac{-\alpha_1 a_1}{2} - \sqrt{(\frac{\alpha_1 a_1}{2})^2 + \frac{1}{k_1}} \qquad \ldots(27)$$

$$m_2 = \frac{-\alpha_1 a_1}{2} + \sqrt{(\frac{\alpha_1 a_1}{2})^2 + \frac{1}{k_1}} \qquad \ldots(28)$$

$$n_1 = \frac{-\alpha_2 a_2}{2} - \sqrt{(\frac{\alpha_2 a_2}{2})^2 + \frac{1}{k_2}} \qquad \ldots(29)$$

$$n_2 = \frac{-\alpha_2 a_2}{2} + \sqrt{(\frac{\alpha_2 a_2}{2})^2 + \frac{1}{k_2}} . \qquad \ldots(30)$$

The arbitrary constants in solutions (25) and (26) can be determined using conditions of velocity and shear stress continuity at the interface, and the no-slip conditions on the walls, namely conditions (3), (4) and (5), in (25) and (26). The following linear equations for the arbitrary constants are thus obtained:

$$c_1 + c_2 - c_3 - c_4 = \frac{C}{\mu_0}\{k_1 \exp[-\alpha_1 b_1] - k_2 \exp[-\alpha_2 b_2]\} \qquad \ldots(31)$$

$$c_1 + c_2 \exp(m_2 - m_1)L_1 = \frac{C}{\mu_0} k_1 \exp[-m_1 L_1 - \alpha_1(a_1 L_1 + b_1)] \qquad \ldots(32)$$

$$c_3 + c_4 \exp(n_2 - n_1)L_2 = \frac{C}{\mu_0} k_2 \exp[-n_1 L_2 - \alpha_2(a_2 L_2 + b_2)] \qquad \ldots(33)$$

and

$$c_2[1 - \exp(m_2 - m_1)L_1] - c_4[1 - \exp(n_2 - n_1)L_2] = \frac{C}{\mu_0} k_2 \exp[-\alpha_2 b_2] * \\ \{\exp[-n_1 L_2]\exp[-\alpha_2 a_2 L_2] - 1\} - \frac{C}{\mu_0} k_1 \exp[-\alpha_1 b_1]\{\exp[-m_1 L_1]\exp[-\alpha_1 a_1 L_1] - 1\} . \qquad \ldots(34)$$

Solution to the linear system of equations, (31)-(34), takes the following forms (in the order they are evaluated):

$$c_4 = B_2 \frac{[A_3 - A_4]}{[A_1 - A_2]} + B_1 \frac{[A_5 - A_6]}{[A_1 - A_2]} \qquad \ldots(35)$$

$$c_2 = A_1 c_4 + B_2 A_4 - B_1 A_5 \qquad \ldots(36)$$

$$c_1 = -c_2 B_3 + B_1 B_4 \qquad \ldots(37)$$

$$c_3 = c_1 + c_2 - c_4 + B_2 - B_1 \qquad \ldots(38)$$

where

$$A_1 = \frac{\{n_2 - n_1 \exp(n_2 - n_1)L_2\}}{\{m_2 - m_1 \exp(m_2 - m_1)L_1\}} \qquad \ldots(39)$$

$$A_2 = \frac{[1 - \exp(n_2 - n_1)L_2]}{[1 - \exp(m_2 - m_1)L_1]} \qquad \ldots(40)$$

$$A_3 = \frac{\{\exp[-n_1 L_2 - \alpha_2 a_2 L_2] - 1\}}{[1 - \exp(m_2 - m_1)L_1]} \qquad \ldots(41)$$

$$A_4 = \frac{\{n_1 \exp[-n_1 L_2 - \alpha_2 a_2 L_2] + \alpha_2 a_2\}}{\{m_2 - m_1 \exp(m_2 - m_1)L_1\}} \qquad \ldots(42)$$

$$A_5 = \frac{\{m_1 \exp[-m_1 L_1 - \alpha_1 a_1 L_1] + \alpha_1 a_1\}}{\{m_2 - m_1 \exp(m_2 - m_1)L_1\}} \qquad \ldots(43)$$

$$A_6 = \frac{\{\exp[-m_1 L_1 - \alpha_1 a_1 L_1] - 1\}}{[1 - \exp(m_2 - m_1) L_1]} \qquad \ldots(44)$$

$$B_1 = \frac{Ck_1 \exp[-\alpha_1 b_1]}{\mu_0} \qquad \ldots(45)$$

$$B_2 = \frac{Ck_2 \exp[-\alpha_2 b_2]}{\mu_0} \qquad \ldots(46)$$

$$B_3 = \exp(m_2 - m_1) L_1 \qquad \ldots(47)$$

and

$$B_4 = \exp[-m_1 L_1 - \alpha_1 a_1 L_1]\,. \qquad \ldots(48)$$

## IV. RESULTS AND DISCUSSION

Results have been obtained for the following choices of parameters.

1) Without loss of generality, select $b_1 = b_2 = 1$ and $\alpha_1 = \alpha_2 = 1$.
2) Select $\frac{C}{\mu_0} = -1$ and $\frac{C}{\mu_0} = -10$ .
3) For each value of $\frac{C}{\mu_0}$ the following combinations of layer thicknesses and viscosity coefficients are chosen:
   $L_1 = -1,\ L_2 = 1, a_1 = -1, a_2 = 1$
   $L_1 = -1, L_2 = 2, a_1 = -1, a_2 = 0.5$
   $L_1 = -2, L_2 = 1, a_1 = -1, a_2 = 2$
4) For each of the selections in 3), above, a subset of the following permeability values are selected:
   $k_1 = 1, k_2 = 1$
   $k_1 = 1, k_2 = 0.1$
   $k_1 = 0.1, k_2 = 1$
   $k_1 = 0.01, k_2 = 0.1$

For the above values of parameters, the arbitrary constants $c_1$, $c_2$, $c_3$ and $c_4$ have been calculated using (35)-(38). The computed values are used in the computations of velocity and shear stress at the interface, and the determination of velocity profiles across the layers. Velocity at the interface, *y = 0*, is obtained using either of the following expressions, obtained from (25) and (26), respectively:

$$u_1(0) = c_1 + c_2 - \frac{C}{\mu_0} k_1 \exp(-\alpha_1 b_1) \qquad \ldots(49)$$

$$u_2(0) = c_3 + c_4 - \frac{C}{\mu_0} k_2 \exp(-\alpha_2 b_2)\,. \qquad \ldots(50)$$

Shear stress at the interface is obtained using either of the following expressions, obtained from (25) and (26), respectively:

$$\frac{du_1}{dy}(0) = c_1 m_1 + c_2 m_2 - \frac{C}{\mu_0} k_1 (m_1 + m_2) \exp(-\alpha_1 b_1) \qquad \ldots(51)$$

$$\frac{du_2}{dy}(0) = c_3 n_1 + c_4 n_2 - \frac{C}{\mu_0} k_2 (n_1 + n_2) \exp(-\alpha_2 b_2)\,. \qquad \ldots(52)$$

Values of the velocity and shear stress at the interface, for the selected combination of parameters, are listed in **Tables 1** and **2**. Both the velocity and shear stress at the interface are influenced by the permeability, driving pressure gradients, layer thicknesses and coefficients of *y* in the viscosity function.

Effects of permeability and pressure gradients on the interfacial velocity and shear stress are illustrated in **Table 1**, which shows the increase in these quantities with pressure gradient, for a given permeability. For a fixed pressure gradient, interfacial velocity and shear stress increase with permeability. In addition, they increase with increasing permeability in either of the layers when the permeability of the other layer remains unchanged. With increasing permeability there is an increase of momentum transfer from one layer to another, thus influencing the velocity in the region near the interface.

Since layer thicknesses and coefficients of *y* must satisfy $\alpha_1 a_1 L_1 = \alpha_2 a_2 L_2$, then for $\alpha_1 = \alpha_2$, the effects of these parameters is illustrated for the combinations shown in **Table 2**, which shows an increase in the velocity and shear stress at the interface with decreasing lower layer thickness. This may be attributed to the lesser influence the upper layer has on the lower layer when the lower layer is of greater thickness.

| | Velocity at Interface | Shear stress at Interface |
|---|---|---|
| $\frac{C}{\mu_0} = -1,\ k_1 = k_2 = 1$ | 0.2642485855 | 0.3551029315 |
| $\frac{C}{\mu_0} = -10,\ k_1 = k_2 = 1$ | 2.642485855 | 3.551029315 |
| | | |
| $\frac{C}{\mu_0} = -1,\ k_1 = 0.01, k_2 = 0.1$ | 0.01692744072 | 0.1429570892 |
| $\frac{C}{\mu_0} = -10,\ k_1 = 0.01, k_2 = 0.1$ | 0.1692744072 | 1.429570892 |
| | | |
| $\frac{C}{\mu_0} = -1,\ k_1 = 1, k_2 = 0.1$ | 1.060520029 | 1.856567696 |
| $\frac{C}{\mu_0} = -1,\ k_1 = 0.1, k_2 = 1$ | 0.0914581620 | 0.2455583205 |

**Table 1: Velocity and shear stress at interface for different $\frac{C}{\mu_0}$ and permeabilities**

$$b_1 = b_2 = 1,\ \alpha_1 = \alpha_2 = 1,\ L_1 = -1\ L_2 = 1,\ a_1 = -1\ a_2 = 1$$

| | Velocity at Interface | Shear stress at Interface |
|---|---|---|
| $L_1 = -1\ L_2 = 1, a_1 = -1\ a_2 = 1$ | 0.2642485855 | 0.3551029315 |
| $L_1 = -1\ L_2 = 2,\ a_1 = -1,\ a_2 = 0.5$ | 1.003176454 | 1.748439545 |
| $L_1 = -2\ L_2 = 1,\ a_1 = -1\ a_2 = 2$ | 0.1956139595 | 0.1174141298 |

**Table 2: Velocity and shear stress at interface for different $L_1, L_2,\ a_1$ and $a_2$**

$$b_1 = b_2 = 1,\ \alpha_1 = \alpha_2 = 1,\ k_1 = k_2 = 1,\ \frac{C}{\mu_0} = -1$$

Viscosity expressions are given by equations (15) and (16). It is clear that the most important parameter in each profile is the coefficient of *y*. The effects of $b_1$ and $b_2$ are manifested in magnifying the viscosity distribution. Therefore, we set $b_1 = b_2 = \alpha_1 = \alpha_2 = 1$ and illustrate the effects of $a_1$ and $a_2$ on the viscosity profiles, through graphing $\frac{\mu_1(y)}{\mu_0}$ and $\frac{\mu_2(y)}{\mu_0}$ versus *y*, for different parameters. The values of $L_1 = -1$, $L_2 = 1$, $a_1 = -1, -2, -5$ and $a_2 = 0.5, 1, 2$ are selected for illustrations in **Figs**. **2** and **3**. These figures show the exponential decrease in viscosity as we move away from the solid boundary towards the interface for all the parameters tested. The choice of viscosity functions produce a continuous viscosity stratification. It is emphasized here that $a_1$ must be negative and $a_2$ positive in order to produce viscosity variations such that the wall viscosities are highest.

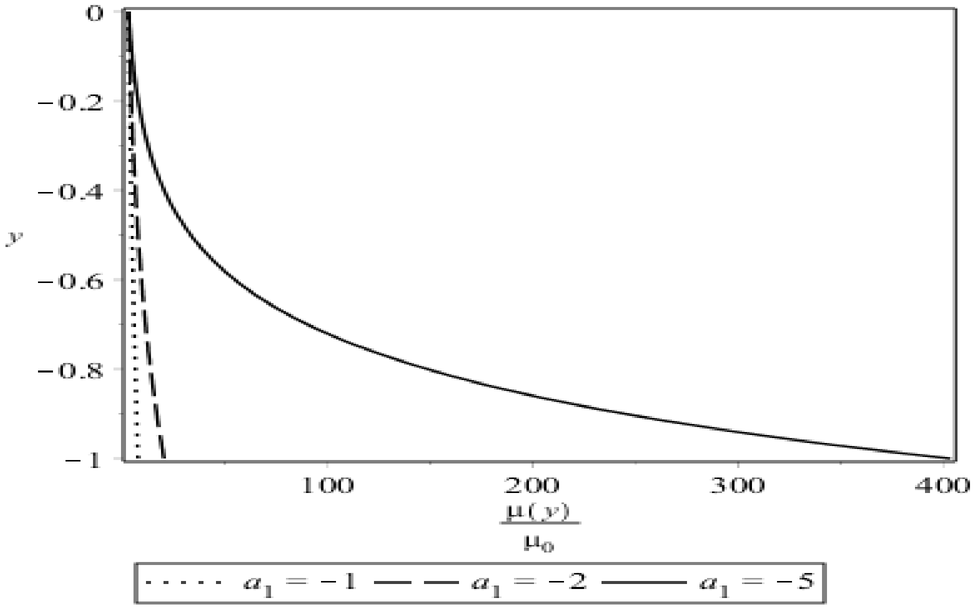


**Fig. 2: Viscosity Profile in the Lower Layer**

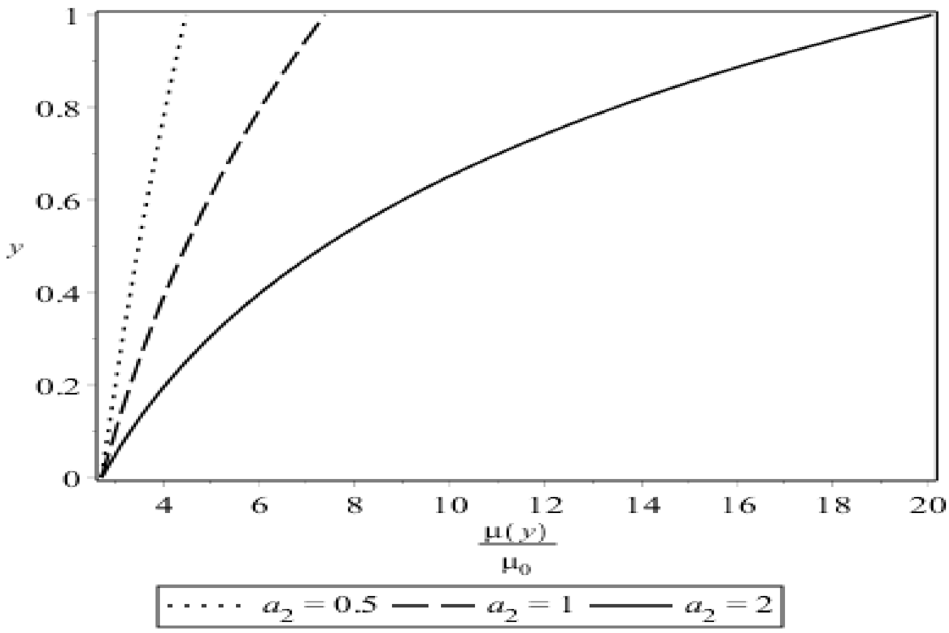


**Fig. 3: Viscosity Profile in the Upper Layer**

Velocity solutions (25) and (26) are represented in **Figs**. **4** to **7** for various values of parameters tested. In **Figs**. **4** and **5**, the velocity profiles are shown for $\frac{C}{\mu_0} = -1$, while **Figs. 6** and **7** show the profiles for $\frac{C}{\mu_0} = -10$. These figures illustrate the changes in the profiles with permeability, and the effect of increasing $a_2$ on decreasing the velocity at the interface.

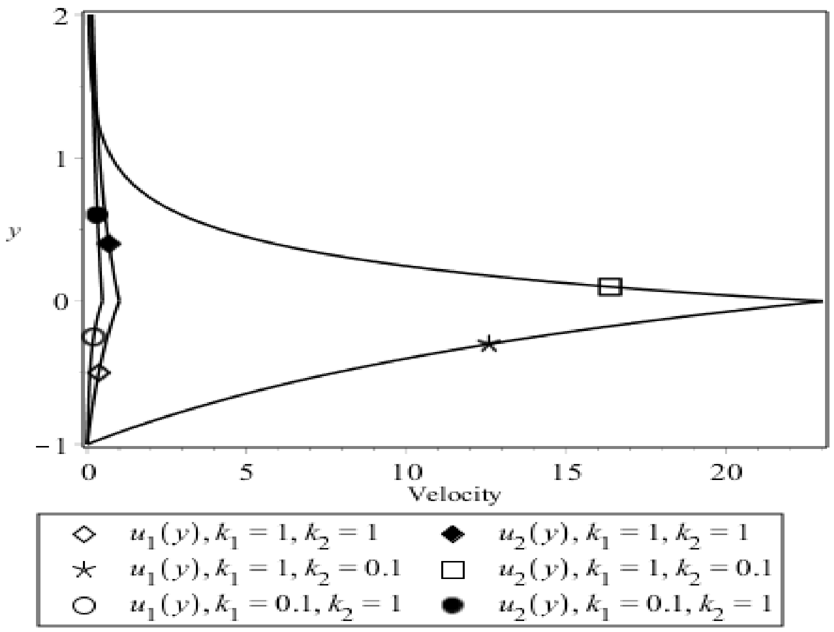


**Fig. 4: Velocity profiles when** $L_1 = -1$ **,** $L_2 = 2$ **,** $a_1 = -1, a_2 = 0.5$ **and** $\frac{C}{\mu_0} = -1$

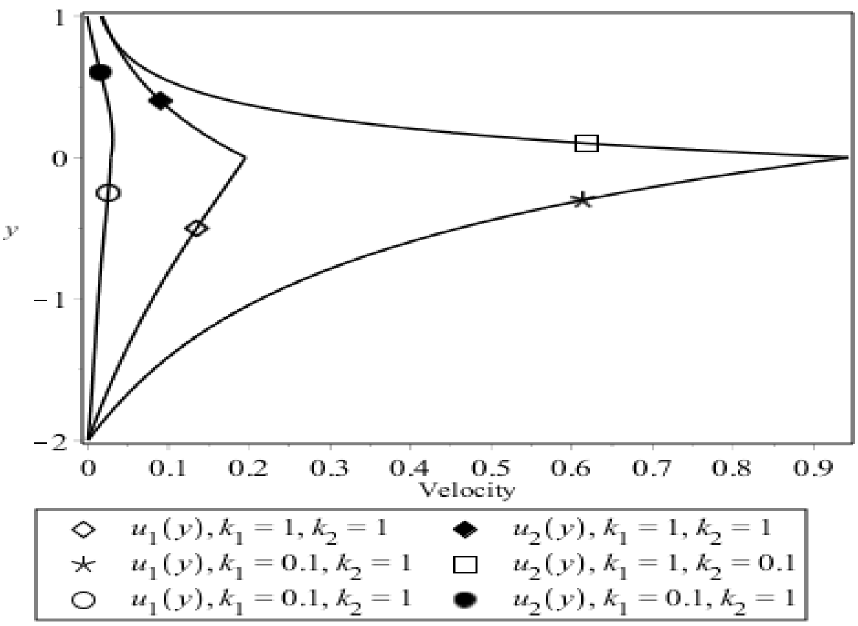


**Fig. 5: Velocity profiles when** $L_1 = -2$ **,** $L_2 = 1$ **,** $a_1 = -1, a_2 = 2$ **and** $\frac{C}{\mu_0} = -1$

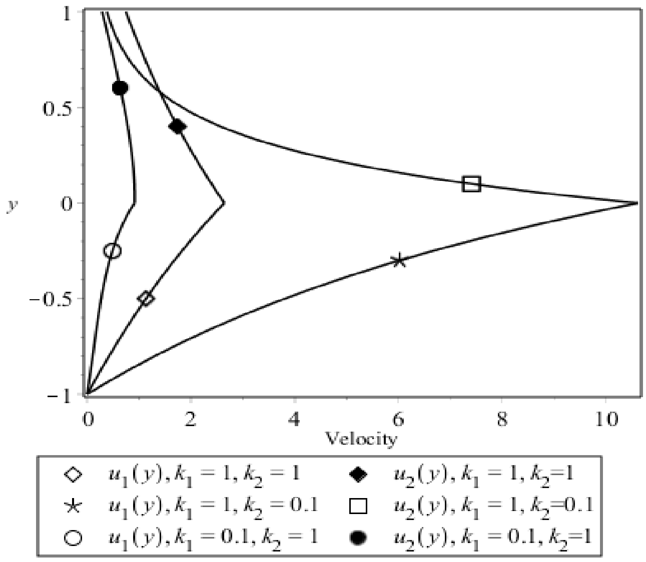


**Fig. 6: Velocity profiles when** $L_1 = -1$, $L_2 = 1$, $a_1 = -1, a_2 = 1$ **and** $\dfrac{C}{\mu_0} = -10$

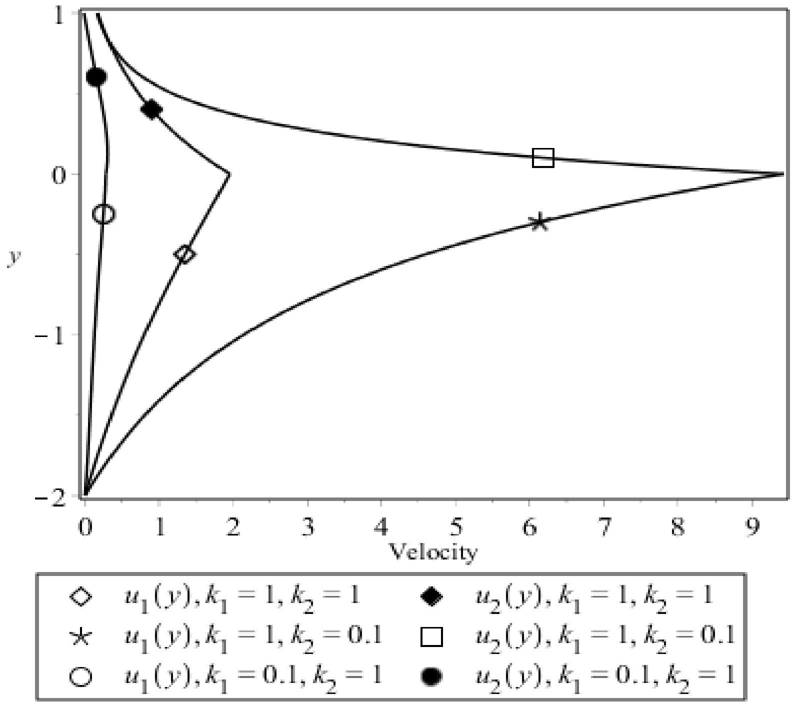


**Fig. 7: Velocity profiles when** $L_1 = -2$, $L_2 = 1$, $a_1 = -1, a_2 = 2$ **and** $\dfrac{C}{\mu_0} = -10$

## V. CONCLUSION

In this work, we formulated and solved the problem of coupled parallel flow through two porous layers when the fluid viscosities are stratified. Appropriate interfacial conditions have been stated, and exponential variations in viscosity have been considered to produce the necessary high wall viscosities. Effects of permeability, pressure gradient and other flow parameters have been discussed. This work initiates and sets the stage for future analysis of coupled parallel flow of fluids with pressure-dependent and temperature-dependent variable viscosities.

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